EUF 2012 - 2sem 23/03/2016

Q1.

a) $\frac{\partial \rho}{\partial t} = 0 \Rightarrow \int J\,ds = I$

$\int J\,ds = \frac{d}{dt}\int \rho\,dv \Rightarrow \int J\,ds = \int \frac{d\rho}{dt}\,dv$

$\int J\,ds = \frac{dQ}{dt}$

$J = \frac{\sigma E}{\ell}$

$\Rightarrow \nabla \cdot J - \frac{\partial \rho}{\partial t} = 0 \Rightarrow \frac{\sigma}{\ell}\nabla D - \frac{\partial \rho}{\partial t} = 0 \Rightarrow \frac{\sigma}{\ell}\rho - \frac{\partial \rho}{\partial t} = 0 \Rightarrow \boxed{\rho = 0}$

b) $\nabla^2 V = \rho \Rightarrow \nabla^2 V = 0$ (Laplace)

$\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0 \Rightarrow V(r) = A\ln r + B$

$r = a \Rightarrow V(a) = V_0 = A\ln a + B$

$r = b \Rightarrow V(b) = 0 = A\ln b + B$

$\therefore B = -A\ln b = -\frac{V_0 \ln(b)}{\ln(\frac{a}{b})}$

$A = \frac{V_0}{\ln(\frac{a}{b})}$

$V(r) = \frac{V_0 \ln r}{\ln(\frac{a}{b})} - \frac{V_0 \ln(b)}{\ln(\frac{a}{b})} = \frac{V_0(\ln(r) - \ln(b))}{\ln(\frac{a}{b})}$

$\boxed{V(r) = \frac{V_0 \ln(\frac{r}{b})}{\ln(\frac{a}{b})}} \Rightarrow \vec{E} = -\nabla V = -\frac{V_0}{r}\cdot\frac{1}{b}\cdot\frac{1}{\ln(\frac{a}{b})} \Rightarrow \boxed{E(r) = \frac{-V_0}{r\ln(\frac{a}{b})}}$

c) $\int E\,ds = \frac{Q}{\varepsilon_0} \Rightarrow E(a)\Big|_0^\ell \Rightarrow \rho_s =$

$Q = \varepsilon_0 \int_0^\ell \frac{-V_0}{a\ln(\frac{a}{b})}\,\underbrace{2\pi a}_{\text{área superficie}}\,d\ell = -\frac{2\pi V_0 \ell \varepsilon}{\ln(\frac{a}{b})} \Rightarrow \frac{C}{\ell} = \frac{Q}{\ell V(r=a)} = -\frac{2\pi\varepsilon}{\ln(\frac{a}{b})}$

$\sim \frac{Q}{\ell V_0}$

d) $\vec{J} \cdot \sigma \vec{E} = -\dfrac{V_0 \sigma}{r \ln\left(\frac{a}{b}\right)}$

$$I = \int J\,dS = \int_a^b \frac{-V_0\sigma}{r\ln\left(\frac{a}{b}\right)} \cdot 2\pi r\,dr = -\frac{V_0\sigma}{\ln\left(\frac{a}{b}\right)} \cdot (b-a) = \frac{V_0\sigma(a-b)}{\ln\left(\frac{a}{b}\right)}$$

$$R = \frac{V_0}{I} = \frac{\ln\left(\frac{a}{b}\right)}{\sigma(a-b)}$$

Q2. $J = \hat{r}\,\dfrac{J_0}{r}\,\mathrm{sen}\left(\frac{\pi r}{a}\right)$

a) $\int J\,dS = I \Rightarrow \int_0^a \frac{J_0}{r}\,\mathrm{sen}\left(\frac{\pi r}{a}\right) 2\pi r\,dr = I \Rightarrow J_0\left(\frac{a}{\pi}\right)\cdot\left(-\cos\left(\frac{\pi r}{a}\right)\right)\Big|_0^a = \frac{I}{2\pi}$

$$-J_0\left[\cos\pi - \cos 0\right] = \frac{I}{2\pi}\cdot\frac{\pi}{a} \Rightarrow \boxed{J_0 = \frac{I}{4a}}$$

b) $\int H\,dl = I_{enc} \Rightarrow B = \dfrac{\mu I}{2\pi r} = \boxed{\dfrac{\mu}{2\pi r}\int \frac{I}{4a}\cdot \mathrm{sen}\left(\frac{\pi r}{a}\right) dS}$ ...

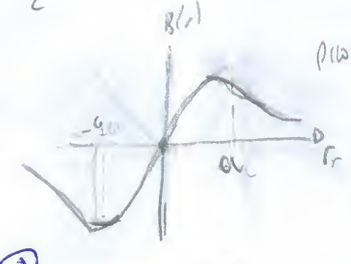
c) $\int B\,dl = \mu_0 I_{enc} \Rightarrow \boxed{B = \dfrac{\mu_0 I\left[1-\cos\left(\frac{\pi r}{a}\right)\right]}{4\pi r}}$

$$I_{enc} = \int_0^r J\,dS = \frac{I}{4a}\int_0^r \frac{1}{r}\,\mathrm{sen}\left(\frac{\pi r}{a}\right)\cdot 2\pi\,dr = \frac{I}{4a}\cdot\left(\frac{a}{\pi}\right)\left[-\cos\left(\frac{\pi r}{a}\right)+1\right]\cdot 2\pi$$

$$= \frac{I}{2}\left[1-\cos\left(\frac{\pi r}{a}\right)\right]$$

d) $B(0) = 0$

$B(a) = \dfrac{\mu_0 I}{2\pi a}$



$$(4\pi r)\cdot(\cos\pi)\cdot\frac{\pi}{a}\cdot\mathrm{sen}\left(\frac{\pi r}{a}\right) - \left[\cos\pi\left(1-\cos\left(\frac{\pi r}{a}\right)\right)\cdot 4\pi\right] = 0$$

$$\left(\frac{r\pi}{a}\,\mathrm{sen}\left(\frac{\pi r}{a}\right)\right) - \left(4 - 4\cos\left(\frac{\pi r}{a}\right)\right) = 0$$

EUF 2018-2ème 23/03/2018

Q3

a) $p = \frac{h}{\lambda}$ $n\lambda = 2\pi r$ (cercle)

$\frac{nh}{p} = 2\pi r \Rightarrow \boxed{nh = m_e v \cdot r}$ ✓

$v = \frac{nh}{m_e r} \Rightarrow$

b) $\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{m}{r}\left(\frac{nh}{m_e r}\right)^2 \Rightarrow \boxed{r_n = \frac{4\pi\varepsilon_0 n^2 h^2}{e^2 m_e}}$ ✓

$E = \frac{1}{2}m\left(\frac{nh}{m_e r}\right)^2 - \frac{e^2}{4\pi\varepsilon_0 r} = \frac{1}{2m_e}\cdot\frac{n^2h^2 \cdot e^4 m_e^2}{(4\pi\varepsilon_0 n^2 h^2)^2} - \frac{e^2 \cdot e^2 m_e}{(4\pi\varepsilon_0)^2 n^2 h^2}$

$E = \frac{1}{2}\frac{e^4 m_e}{(4\pi\varepsilon_0)^2 n^2 h^2} - \frac{e^4 m_e}{(4\pi\varepsilon_0)^2 n^2 h^2} = \boxed{-\frac{e^4 m_e}{8(\pi\varepsilon_0)^2 n^2 h^2}}$ ✓

c) $E_1 = +13{,}6$ 17,6 150 1,5 13,6

$\boxed{E_3 = \frac{E_1}{n^2} = \frac{-13{,}6}{9} = -1{,}5\ eV}$

d) $E_2 = -3{,}6\ eV$ $\Rightarrow \Delta E = 2{,}75\ eV$

$E_4 = \frac{-13{,}6}{16} = -0{,}85$

$E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{2{,}75} = \frac{4{,}14\cdot10^{-15}\cdot3\cdot10^8}{2{,}75}$

$\lambda = 4{,}14\cdot10^{-7} \Rightarrow \boxed{414\ nm}$

ultraviolet visible

Q4

work required to remove electron

a) $K = h\nu - W$ $\qquad$ $K_{max} = eV_0$

$K_{max} = h\nu - w_0$

work function, min energy needed to remove

$V_0 = \frac{h\nu}{e} - \frac{w_0}{e}$ $\qquad$ slope fig 2 is h (eV)

60 | 16
120 0,37

$$h = \frac{(1-0,4)}{(7-5,4)10^{14}} = \frac{0,6 \cdot 10^{-14}}{1,6} = 3,7 \cdot 10^{-15}\ eV$$

b) $w_0 = h \cdot 4,5 \cdot 10^{14}$ $\quad (V_0 = 0)$

$$\boxed{w_0 = 4 \cdot 4,5 \cdot 10^{-1} = 1,8\ eV}$$

c) $K_{max} = 4 \cdot 10^{-15} \cdot 10^{15} - 1,8 = 2,2\ eV$

d) classicamente 0 ?

neo prevê o aumento da freq inversa da intensidade mudar a energia e tb tem o limite mínimo de freq

Q5.

a) $T_f = \frac{T_2 + T_1}{2}$ $\rightarrow$ $-Q_A = -mc(T_f - T_A) = mc(T_f - T_B) = Q_B$

$2T_f = T_A + T_B$

$\Delta T_A = ($

$\Delta S = \Delta S_A + \Delta S_B$

$= C_p\left[\ln\left(\frac{T_f^2}{T_1 T_2}\right)\right]$

$\Delta S = \int_{T_i}^{T_f} \frac{mc\,dT}{T}$

b) $\frac{C_p}{T} = \frac{\partial S}{\partial T} = \Delta S = C_p \ln\left(\frac{T_f}{T_i}\right)$ $\quad$ or $dQ = \frac{mc\,dT}{T} \Rightarrow dS = \frac{dQ}{T} \Rightarrow$

$\Delta S_A = C_p \ln\left(\frac{T_f}{T_1}\right)$ $\quad \Delta S_B = C_p \ln\left(\frac{T_f}{T_2}\right)$

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c) Proceso reversible
estado inicial = estado final $\Delta U = 0$ $Q_1 = W + Q_2$

$S_A = \frac{Q_A}{T_1} = \frac{Q_B}{T_2} = S_B \Rightarrow \Delta S = 0$

$\int_{T_2}^{T_1} \frac{Q_2}{T_2} - \int_{T_1}^{T_1} \frac{Q_1}{T_1} \geq 0 \Rightarrow \int_{T_2}^{T_1} \frac{c_p dT}{T_2} - \int_{T_1}^{T_1} \frac{c_p dT}{T_1} \geq 0 \Rightarrow \ln\left(\frac{T_1^2}{T_1 T_2}\right) \geq 0$

$\frac{T_1^2}{T_1 T_2} \geq 1 \Rightarrow \boxed{T_1 \geq \sqrt{T_1 T_2}}$

d) $W = Q_2 - Q_1$

$= c_p \Delta T_A - c_p \Delta T_B = c_p [(T_1 - T_1) - (T_1 - T_2)] \Rightarrow$

$\boxed{W = c_p (T_2 + T_2 - 2T_1)}$

Q6

a) $L = \frac{1}{2}\mu \dot{r}^2 + \frac{1}{2}\mu r^2 \dot{\theta}^2 - U(r) \rightarrow \boxed{\frac{d^2 U}{d\theta^2} + U = -\frac{\mu}{l^2 U^2} F\left(\frac{1}{U}\right)}$

$\frac{\alpha}{r} = 1 + \varepsilon \cos\theta$

$\varepsilon > 1$ hyperbola $\Rightarrow \varepsilon > 0$

$0 < \varepsilon < 1$ (ellipse planetary motion)

$L = r_1 \times m v$

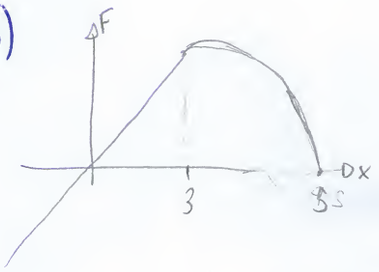
$= r_0 \cdot m \cdot v_0 \sin 30°$

Q7. a) $F = -Kx \Rightarrow U = \frac{mw^2x^2}{2} \Rightarrow \frac{1}{2}mv^2 = \frac{450g \cdot w^2 \cdot 1cm^2}{2}$

$w^2 = \frac{K}{m}$

$10^{-3} = \frac{0,450 \cdot w^2 \cdot 10^{-4}}{2} \Rightarrow w = \sqrt{\frac{20}{0,450}}$ $\quad K = w^2 m$

b)



c) $\frac{Kx^2}{2} = \frac{mv_{max}^2}{2} \Rightarrow v_{max}^2 = \frac{K(3 \cdot 10^{-2})^2}{m}$

d) $\frac{Kx^2}{2} + \frac{mv^2}{2} = \frac{mv_{max}^2}{2} \Rightarrow \frac{K(2 \cdot 10^{-2})^2}{2} + \frac{mv^2}{2} = \frac{mv_{max}^2}{2}$

e) $\frac{Kx^2}{2} = \frac{K(5 \cdot 10^{-2})^2}{2}$ $\quad x = ?$

EUF 2012-2sm 26/03/2016

Q8.

a) $\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} = E\psi(x)}$ $0<x<a$

b) $\frac{\partial^2\psi(x)}{\partial x^2} - k^2\psi(x) = 0 \Rightarrow k^2 = \frac{2mE}{\hbar^2} \Rightarrow \boxed{E_n = \frac{n^2\pi^2\hbar^2}{a^2 2m}}$

$\psi(x) = A\cos(kx) + B\sin(kx)$

$x=0=a \Rightarrow \psi=0 \Rightarrow \psi(x) = B\sin\left(\frac{n\pi}{a}\cdot x\right) \Rightarrow \boxed{\psi_n = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)}$

$k = \frac{n\pi}{a}$

$\int_0^a |\psi(x)|^2 = 1 \Rightarrow \boxed{B = \sqrt{\frac{2}{a}}}$

c) $\frac{\partial^2\psi(x)}{\partial x^2} + \psi(x)\left(W_0\sin\left(\frac{\pi x}{a}\right) - k^2\right) = 0$ $|x|<<1$ $(1+x)^k = 1+\alpha x$

estado fundamental

$E_n^{(1)} = \int_0^a W_0\sin\left(\frac{\pi x}{a}\right)\cdot\frac{2}{a}\sin^2\left(\frac{\pi x}{a}\right)dx$ $\boxed{E_n^{(1)} = \langle\psi_n^0|H'|\psi_n^0\rangle}$

$E_n^{(1)} = \frac{2}{a}\left[\int_0^a \frac{W_0\sin\left(\frac{\pi x}{a}\right)}{2} - \frac{W_0\overbrace{\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{2\pi x}{a}\right)}^{\frac{1}{2}\sin\left(\frac{3\pi x}{a}\right) - \sin\left(\frac{\pi x}{a}\right)}}{2}\right]dx$

$= \frac{2}{a}\left[\frac{W_0}{2}\cdot\left(\frac{-2a}{\pi}\right) - \right.$ $\boxed{E_n = E_n^{(0)} + E_n^{(1)}}$

7

Q9. a) $\hat{n}\cdot\vec{\sigma} = \sigma_x \sin\theta\cos\phi + \sigma_y \sin\theta\sin\phi + \sigma_z\cos\theta$

$$= \begin{pmatrix}0&1\\1&0\end{pmatrix}\sin\theta\cos\phi + \begin{pmatrix}0&-i\\i&0\end{pmatrix}\sin\theta\sin\phi + \begin{pmatrix}1&0\\0&-1\end{pmatrix}\cos\theta$$

$$S_n = \frac{\hbar}{2}\begin{pmatrix}\cos\theta & e^{-i\phi}\sin\theta \\ e^{i\phi}\sin\theta & -\cos\theta\end{pmatrix}$$

b) $(\cos\theta-\lambda)(-\cos\theta-\lambda) - \sin^2\theta = 0$

$\lambda^2 - 1 = 0 \Rightarrow \boxed{\lambda = \pm 1}$ $\frac{1}{2}$

c) $\frac{\hbar}{2}\begin{bmatrix}\cos\theta-1 & e^{-i\phi}\sin\theta \\ \cdots & \cdots\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix} = 0$

$\sin^2(\frac{\theta}{2}) = \frac{1}{2} - \frac{1}{2}\cos(\theta)$

$\cos(\frac{\theta}{2}) = \sqrt{\frac{1+\cos\theta}{2}}$

$2\sin^2(\frac{\theta}{2})$

$(\cos\theta - 1)a + e^{-i\phi}\sin\theta\, b = 0$ a

$\left(\frac{\hbar}{2} + 1 - \cos\theta\right)a = e^{-i\phi}\sin\theta\, b \Rightarrow \begin{bmatrix}\cos(\frac{\theta}{2})\\ e^{i\phi}\sin(\frac{\theta}{2})\end{bmatrix}$

$\left(\frac{\hbar}{2} + 2\sin^2\left(\frac{\theta}{2}\right)\right)a = e^{-i\phi}\sin\theta\, b$

$\frac{\hbar}{2}\cos(\frac{\theta}{2}) + 2\sin^2(\frac{\theta}{2})\cdot\cos(\frac{\theta}{2}) = e^{-i\phi}\sin\theta\, b$

$\frac{\hbar}{2}\cos(\frac{\theta}{2}) + 2\cdot\sin(\frac{\theta}{2})\cdot\frac{1}{2}\cdot\sin\theta = e^{-i\phi}\sin(\theta)\, b$

$\boxed{b = e^{i\phi}\sin\left(\frac{\theta}{2}\right)}$

EuF 2012. 2sem 26/33

d) $\theta = 60°$ $\quad \phi = 45°$ $\quad \chi_+ = \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)\cdot\frac{1}{2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \sqrt{3} \\ \sqrt{2}(1+i) \end{bmatrix}$

$e^{i\frac{\pi}{4}} = \cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)$

$= \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$

$\langle S_{\hat{n}}^+ | S_z^+ \rangle = \frac{1}{2}\begin{bmatrix} \sqrt{3} & \sqrt{2}(1-i) \end{bmatrix}\cdot\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{\sqrt{3}}{2}$

$P(S_{\hat{n}}^+) = |\langle S_{\hat{n}}^+ | S_z^+ \rangle|^2 = \frac{3}{4}$

?

$\left(\frac{k_B T}{c}\right) \int_{-\infty}^{\infty} e^{-x^2} \cdot \left(\frac{\pi}{\alpha}\right)^{\frac{1}{2}}$

$E = cp$

Q12

a) $\varepsilon = cp$. $\quad Z = \iiint_V \iiint_{-\infty}^{\infty} e^{-\beta c(p_x + p_y + p_z)}\, dp_x\, dp_y\, dp_z\, dV$

$Z = \frac{1}{N!}\frac{V^N}{h^{3N}} \int d^3p\, e^{-\beta|p|c} = \frac{1}{N!}\frac{V^N}{h^{3N}} \cdot \frac{4\pi}{(\beta c)^2} \int_0^{\infty} dp\, e^{-\beta c p} = \frac{1}{N!}\frac{(8\pi V)^N}{\left(\frac{hc}{k_B T}\right)^{3N}}$

b) $P = -\left(\frac{\partial F}{\partial V}\right)_T \Rightarrow F = U - TS = -k_B T \ln Z$

c) $F = U - TS \Rightarrow S = \frac{U - F}{T} \Rightarrow U = k_B T^2 \frac{d \ln Z}{dp}$ ou $S = -\left(\frac{\partial F}{\partial T}\right)_V$

(9)